Adı Soaydı:
Öğr. No: 28.05.2012

İNŞAAT MÜHENDİSLİĞİ 2. SINIF MÜHENDİSLİK MATEMATİĞİ FİNAL

1. a) Aşağıdaki verilen eğrinin uzunluğunu bulunuz

$$x = t^3, \quad y = 3t^2/2, \quad 0 \le t \le \sqrt{3}$$

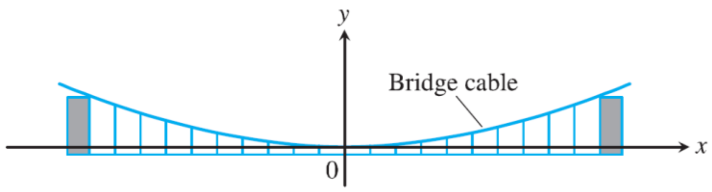
b) Aşağıdaki eğrinin ağırlık merkezi koordinatlarını bulunuz

$$x = \cos t + t \sin t, \quad y = \sin t - t \cos t, \quad 0 \le t \le \pi/2.$$

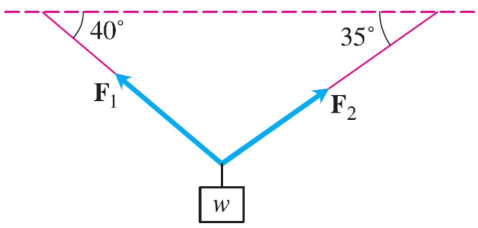
2. Şekilde görülen asma köprünün kabloları her yatay metre başına *w*-newtonluk düzgün yük taşımaktadır. Eğer kablonun orijindeki yatay gerilimi H ise kablo eğrisinin

$$\frac{dy}{dx} = \frac{w}{H}x$$

denklemini sağlamaktadır. x=0 iken y=0 başlangıç koşuluna sahip bu denklemi çözerek kablonun bir parabol şeklinde olduğunu gösteriniz.



3. **a)** Aşağıdaki şekilde verildiği gibi *w*-newtonluk ağırlığın iki telle asılı olduğuna göre, F_2=100 N ise *w* ağırlığını ve F_1 vektörünün büyüklüğünü bulunuz.



b) Aşağıdaki vektörler için (u x v).w = (v x w).u = (w x u).v eşitliğini doğrulayınız ve u, v, w vektörlerinin belirlediği paralelyüzlünün (kutu) hacmini bulunuz.

u = i – j + k,   v = 2i + j + 2k,   w = – i + 2j – k

4. Aşağıdaki fonksiyonun $\frac{\partial f}{\partial x}$ ve $\frac{\partial f}{\partial y}$ kısmi türevlerini bulunuz.

$$f(x, y) = \frac{2y}{y + \cos x}$$

5. Aşağıdaki çift katlı integrallerin integrasyon bölgesini çiziniz ve integralini hesaplayınız

a) $\displaystyle\int_0^{\pi}\int_0^{x} x \sin y \, dy \, dx$ **b)** $\displaystyle\int_0^{\pi}\int_0^{\sin x} y \, dy \, dx$

Süre: 100 dak

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CEVAPLAR

Cevap 1(a,b, s626).

26. $\frac{dx}{dt} = 3t^2$ and $\frac{dy}{dt} = 3t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(3t^2)^2 + (3t)^2} = \sqrt{9t^4 + 9t^2} = 3t\sqrt{t^2+1}$ $\left(\text{since } t \geq 0 \text{ on } \left[0, \sqrt{3}\right]\right)$

$\Rightarrow$ Length $= \int_0^{\sqrt{3}} 3t\sqrt{t^2+1}\, dt$; $\left[u = t^2 + 1 \Rightarrow \frac{3}{2}\, du = 3t\, dt;\ t = 0 \Rightarrow u = 1,\ t = \sqrt{3} \Rightarrow u = 4\right]$

$\rightarrow \int_1^4 \frac{3}{2} u^{1/2}\, du = \left[u^{3/2}\right]_1^4 = (8-1) = 7$

37. Let the density be $\delta = 1$. Then $x = \cos t + t \sin t \Rightarrow \frac{dx}{dt} = t \cos t$, and $y = \sin t - t \cos t \Rightarrow \frac{dy}{dt} = t \sin t$

$\Rightarrow dm = 1 \cdot ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \sqrt{(t \cos t)^2 + (t \sin t)^2} = |t|\, dt = t\, dt$ since $0 \leq t \leq \frac{\pi}{2}$. The curve's mass is

$M = \int dm = \int_0^{\pi/2} t\, dt = \frac{\pi^2}{8}$. Also $M_x = \int \widetilde{y}\, dm = \int_0^{\pi/2} (\sin t - t \cos t)\, t\, dt = \int_0^{\pi/2} t \sin t\, dt - \int_0^{\pi/2} t^2 \cos t\, dt$

$= [\sin t - t \cos t]_0^{\pi/2} - [t^2 \sin t - 2 \sin t + 2t \cos t]_0^{\pi/2} = 3 - \frac{\pi^2}{4}$, where we integrated by parts. Therefore,

$\overline{y} = \frac{M_x}{M} = \frac{\left(3 - \frac{\pi^2}{4}\right)}{\left(\frac{\pi^2}{8}\right)} = \frac{24}{\pi^2} - 2$. Next, $M_y = \int \widetilde{x}\, dm = \int_0^{\pi/2} (\cos t + t \sin t)\, t\, dt = \int_0^{\pi/2} t \cos t\, dt + \int_0^{\pi/2} t^2 \sin t\, dt$

$= [\cos t + t \sin t]_0^{\pi/2} + [-t^2 \cos t + 2 \cos t + 2t \sin t]_0^{\pi/2} = \frac{3\pi}{2} - 3$, again integrating by parts. Hence

$\overline{x} = \frac{M_y}{M} = \frac{\left(\frac{3\pi}{2} - 3\right)}{\left(\frac{\pi^2}{8}\right)} = \frac{12}{\pi} - \frac{24}{\pi^2}$. Therefore $(\overline{x}, \overline{y}) = \left(\frac{12}{\pi} - \frac{24}{\pi^2}, \frac{24}{\pi^2} - 2\right)$.

Cevap 2 (s647):

70. $y = \int \frac{w}{H} x\, dx = \frac{w}{H}\left(\frac{x^2}{2}\right) + C = \frac{wx^2}{2H} + C$; $y = 0$ when $x = 0 \Rightarrow 0 = \frac{w(0)^2}{2H} + C \Rightarrow C = 0$; therefore $y = \frac{wx^2}{2H}$ is the equation of the cable's curve

Cevap 3. (a, b, s673)

47. $\mathbf{F}_1 = \left\langle -|\mathbf{F}_1| \cos 40°, |\mathbf{F}_1| \sin 40° \right\rangle$, $\mathbf{F}_2 = \langle 100 \cos 35°, 100 \sin 35° \rangle$, and $\mathbf{w} = \langle 0, -w \rangle$. Since $\mathbf{F}_1 + \mathbf{F}_2 = \langle 0, w \rangle$

$\Rightarrow \left\langle -|\mathbf{F}_1| \cos 40° + 100 \cos 35°, |\mathbf{F}_1| \sin 40° + 100 \sin 35° \right\rangle = \langle 0, w \rangle \Rightarrow -|\mathbf{F}_1| \cos 40° + 100 \cos 35° = 0$ and

$|\mathbf{F}_1| \sin 40° + 100 \sin 35° = w$. Solving the first equation for $|\mathbf{F}_1|$ results in: $|\mathbf{F}_1| = \frac{100 \cos 35°}{\cos 40°} \approx 106.933$ N. Substituting this result into the second equation gives us: $w \approx 126.093$ N.

$$|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}| = \text{ abs} \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ -1 & 2 & -1 \end{vmatrix} = \mathbf{0}$$

If $\mathbf{u} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{v} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, and $\mathbf{w} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$, then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u} = \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$ and $(\mathbf{w} \times \mathbf{u}) \cdot \mathbf{v} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ which all have the same absolute value, since the interchanging of two rows in a determinant does not change its absolute value $\Rightarrow$ the volume is

Cevap 4 (s767)

Solution We treat f as a quotient. With y held constant, we get

$$f_x = \frac{\partial}{\partial x}\left(\frac{2y}{y + \cos x}\right) = \frac{(y + \cos x)\frac{\partial}{\partial x}(2y) - 2y\frac{\partial}{\partial x}(y + \cos x)}{(y + \cos x)^2}$$

$$= \frac{(y + \cos x)(0) - 2y(-\sin x)}{(y + \cos x)^2} = \frac{2y \sin x}{(y + \cos x)^2}.$$

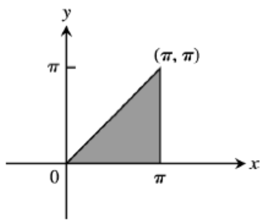
With x held constant, we get

$$f_y = \frac{\partial}{\partial y}\left(\frac{2y}{y + \cos x}\right) = \frac{(y + \cos x)\frac{\partial}{\partial y}(2y) - 2y\frac{\partial}{dy}(y + \cos x)}{(y + \cos x)^2}$$

$$= \frac{(y + \cos x)(2) - 2y(1)}{(y + \cos x)^2} = \frac{2 \cos x}{(y + \cos x)^2}.$$

Cevap 5. (a, b, s848)

19. $\int_0^{\pi}\int_0^{x}(x \sin y)\, dy\, dx = \int_0^{\pi} [-x \cos y]_0^x\, dx$
$= \int_0^{\pi}(x - x \cos x)\, dx = \left[\frac{x^2}{2} - (\cos x + x \sin x)\right]_0^{\pi}$
$= \frac{\pi^2}{2} + 2$



20. $\int_0^{\pi}\int_0^{\sin x} y\, dy\, dx = \int_0^{\pi} \left[\frac{y^2}{2}\right]_0^{\sin x} dx = \int_0^{\pi} \frac{1}{2} \sin^2 x\, dx$
$= \frac{1}{4}\int_0^{\pi}(1 - \cos 2x)\, dx = \frac{1}{4}\left[x - \frac{1}{2} \sin 2x\right]_0^{\pi} = \frac{\pi}{4}$